

# Maximal Nine Dimensional Supergravity, General gaugings and the Embedding Tensor

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We construct the most general maximal gauged/massive supergravity in  $d = 9$  dimensions and determine its extended field content by using the embedding tensor method.

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## 1 Introduction

The embedding-tensor formalism provides a systematic way of finding the extended field content of SUGRA Theories (for recent reviews see Refs. [1] see also Refs. [2–5] and [6–12]). One important feature of this formalism is that it requires the systematic introduction of new higher-rank potentials which are related by Stückelberg gauge transformations (that is, the *tensor hierarchy* of the theory [4, 5, 9, 13–15]). They can be taken as the (bosonic) extended field content of the theory. In Supergravity Theories one may need to take into account additional constraints on the possible gaugings, but, if the gauging is allowed by supersymmetry, then gauge invariance will require the introduction of all the fields in the associated tensor hierarchy and, since gauge invariance is a pre-condition for supersymmetry, the tensor hierarchy will be automatically compatible with supersymmetry. The  $d = 9$  SUGRA theory has (unless their 10D descendants) three vector fields, the embedding-tensor formalism is well suited to study all its possible gaugings and find its extended field content. Gaugings of the maximal  $d = 9$  supergravity have been obtained in the past by generalized dimensional reduction [16, 17]. However, the possible combinations of deformations were not studied and some of the higher-rank fields are associated to the constraints on the combinations of deformations. Furthermore, we do not know if other deformations, with no higher-dimensional origin (such as Romans’ massive deformation of the  $N = 2A, d = 10$  supergravity) are possible. Our goal in this work will be to make a systematic study of all these possibilities using the embedding-tensor formalism plus supersymmetry to identify the extended-field content of the theory, finding the rôle played by the possible 7-, 8- and 9-form potentials, and compare the results with the prediction of the  $E_{11}$  approach.

## 2 Maximal $d = 9$ supergravity: the undeformed theory

There is only one undeformed (*i.e.* ungauged, massless) maximal (*i.e.*  $N = 2$ ) 9-dimensional supergravity [18]. The fundamental (*electric*) fields of this theory are:  $(e_\mu{}^a, \varphi, \tau \equiv \chi + ie^{-\phi}, A^I{}_{(1)}, B^i{}_{(2)}, C_{(3)}, \psi_\mu, \tilde{\lambda}, \lambda)$ .<sup>1</sup> The complex scalar  $\tau$  parametrizes an  $SL(2, \mathbb{R})/U(1)$  coset also described by a symmetric  $SL(2, \mathbb{R})$  matrix  $\mathcal{M}$  (see [19] for full definitions of these and other quantities). Undeformed field strengths,  $F^I, H^i, G$ , of the electric  $p$ -forms are defined which are covariant under a set of abelian gauge transformations. The field strengths follow a set of simple Bianchi and (trivial) Ricci identities ( $ddF's = 0$ ). The bosonic action

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<sup>1</sup> Where  $I = 0, \mathbf{i}$  (indices of the 1- and 6-forms) with  $\mathbf{i}, \mathbf{j}, \mathbf{k} = 1, 2$  and  $i, j, k = 1, 2$  (2- and 5-forms).

and the corresponding equations of motion can be explicitly written. The latter have as global symmetry group  $G \equiv SL(2, \mathbb{R}) \times (\mathbb{R}^+)^2$ .<sup>2</sup> The representation content of the theory is as follows. Under  $SL(2, R)$  transformations:  $(\phi, \tau, C_{(3)}) : \mathbf{1}, B_{(2)} : \mathbf{2}, A_{(1)} : \mathbf{1} + \mathbf{2}$ . The weights of the electric fields under all the scaling symmetries are given in [19]. The dimension of the global symmetry group, which acts on the scalar manifold, is larger than that of the scalar manifold itself. There is one Noether 1-form current  $j_A$  associated to each of the generators of the global symmetries ( $T_A, A = 1 - 5$ ) which are (under  $SL(2, R)$ )  $j_A \sim (j_{\mathbf{3}}, j_{\mathbf{1}}, j_{\mathbf{1}'}).$  These currents are conserved on-shell. The SUSY rules are well known, to lowest order in fermions can be found in [20, 21] (see [19] for corrections).

**Equations of motion (EOMS) and Magnetic fields.** The EOMS for the electrical fields, can be written, after some algebra, in a particularly interesting form for our purposes of defining magnetic duals. They read:

$$\begin{aligned} d\left(e^{\frac{4}{\sqrt{7}}\varphi} \star F^0\right) &= -e^{-\frac{1}{\sqrt{7}}\varphi} \mathcal{M}_{ij}^{-1} F^i \wedge \star H^j + \frac{1}{2} G \wedge G, \\ d\left(e^{\frac{3}{\sqrt{7}}\varphi} \mathcal{M}_{ij}^{-1} \star F^j\right) &= -e^{\frac{3}{\sqrt{7}}\varphi} \mathcal{M}_{ij}^{-1} F^0 \wedge \star H^j + e^{\frac{2}{\sqrt{7}}\varphi} H_i \wedge \star G, \\ d\left(e^{-\frac{1}{\sqrt{7}}\varphi} \mathcal{M}_{ij}^{-1} \star H^j\right) &= e^{\frac{2}{\sqrt{7}}\varphi} F_i \wedge \star G - H_i \wedge G, \\ d\left(e^{\frac{2}{\sqrt{7}}\varphi} \star G\right) &= F^0 \wedge G + \frac{1}{2} H^i \wedge H_i. \end{aligned}$$

It is straightforward from these expressions to define the duality relations, directly for the magnetic field strengths  $(\tilde{G}_{(5)}, \tilde{H}_{(6),i}, \tilde{F}_{(7),I})$  as:

$$\begin{aligned} \tilde{G} &\equiv e^{\frac{2}{\sqrt{7}}\varphi} \star G, & \tilde{F}_0 &\equiv e^{\frac{4}{\sqrt{7}}\varphi} \star F^0, \\ \tilde{H}_i &\equiv e^{-\frac{1}{\sqrt{7}}\varphi} \mathcal{M}_{ij}^{-1} \star H^j, & \tilde{F}_i &\equiv e^{\frac{3}{\sqrt{7}}\varphi} \mathcal{M}_{ij}^{-1} \star F^j. \end{aligned}$$

The relation of the magnetic dual field strengths and dual potentials  $(\tilde{C}_{(4)}, \tilde{B}_{(5)}, \tilde{A}_{(6)})$  is not unique and will be obtained afterwards. Another, slightly different, way to define magnetic fields potentials and identify their field strengths, consists in writing the equations of motion of the  $p$ -forms as total derivatives. One can easily check that both methods lead to the same or compatible results. We complete a full characterization of the dual fields by recursion starting from the lowest levels of the hierarchy. We obtain in this way [19]: their representation content under global transformations (weights,  $SL(2, R)$  representations); explicit, hierarchy-compatible, expressions for the gauge variations; explicit dual field strengths in terms of the potentials and their Susy transformations (possibly modulo dual relations). As usual, the Bianchi identities of the electric fields are the EOMS for the Magnetic Duals while the EOMS of the Electric ones are the Bianchi identities for the Magnetic fields.

**Duals of scalars, and Noether currents.** This dualization procedure is made possible by the gauge symmetries associated to all the  $p$ -form potentials for  $p > 0$  (massless  $p$ -forms with  $p > 0$ ). For the scalars, 0-form fields, there is one Noether 1-form current  $j_A$  associated to each of the generators of the global symmetries of the theory  $T_A$ . These currents are conserved on-shell. We can define a  $(d-2)$ -form potential  $\tilde{A}_{(d-2)}^A$  by  $d\tilde{A}_{(d-2)}^A = J^A \equiv G^{AB} \star j_B$ . Thus, in summary, the dualization procedure indicates that for each electric  $p$ -form with  $p > 0$  there is a dual magnetic  $(7-p)$ -form transforming in the conjugate representation, there are as many magnetic  $(d-2)$ -form duals of the scalars as the dimension of the global group (and not of as the dimension of the scalar manifold) and that they transform in the co-adjoint representation.

**N2-SUGRA D9 boson tensor hierarchy.** We write together the Bianchi identities for the electrical fields, the dual magnetic fields (which are just the equations of motion of the electrical fields) and the dual

<sup>2</sup> One of the  $(\mathbb{R}^+)^2$  scaling transformations (" $\alpha$ ") acts on the metric and only leaves the equations of motion invariant. The second one, " $\beta$ ", leaves invariant both the metric and the action: it is the *trombone symmetry* which may not survive to higher-derivative string corrections.

Noether currents (which are just the conservation equations for the Noether forms) to get the ( 1-8 rank ) hierarchy of bosonic form equations which is shown in table1 (left).

$$\begin{array}{ll}
 dF_\phi = 0, & dF_\tau = 0, \quad \mathcal{D}\vartheta = 0, \quad \mathcal{D}Z^i s = 0, \\
 dF^I = 0, & \mathcal{D}F_\phi = Z_{\phi,I} F^I, \quad \mathcal{D}F_\tau = Z_{\tau,I} F^I, \\
 dH^i + F^0 F^i = 0, & \mathcal{D}F^I = Z_i^I H^i, \\
 dG - F^i H_i = 0, & \mathcal{D}H^i + F^0 F^i = Z^i G, \\
 d\tilde{G} + F^0 G + \frac{1}{2} \epsilon_{ij} H^i H^j = 0, & \mathcal{D}G - F^i H_i = Z \tilde{G}, \\
 d\tilde{H}_i + F_i \tilde{G} - H_i G = 0, & \mathcal{D}\tilde{G} + F^0 G + \frac{1}{2} H^i H_i = Z^i H_i, \\
 d\tilde{F}_0 + F^j \tilde{H}_j - \frac{1}{2} G G = 0, & \mathcal{D}\tilde{H}_i + F_i \tilde{G} - H_i G = Z_i^I \tilde{F}_I^0, \\
 d\tilde{F}_i + F^0 \tilde{H}_i - H_i \tilde{G} = 0, & \mathcal{D}\tilde{F}_0 + F^i \tilde{H}_i - \frac{1}{2} G G = \vartheta_0^A J_A, \\
 dJ_A = 0. & \mathcal{D}\tilde{F}_i + F^0 \tilde{H}_i - H_i \tilde{G} = \vartheta_i^A J_A.
 \end{array}$$

**Table 1** (Left) Undeformed Bianchi hierarchy. We have also added the trivial Bianchi equations corresponding to the 1-form “field strengths” (not related by duality):  $F_\phi \equiv d\phi$ ,  $F_\tau \equiv d\tau$ . (Right) Deformed Bianchi Hierarchy. We also add the conditions which lead to the gauge invariance of the (constant) deformation parameters. Indices are lowered and raised using the antisymmetric tensor  $\epsilon_{ij}$  where appropriate.

### 3 Deforming the maximal $D9$ supergravity

The maximal 9D SUGRA has  $3 \times 1$ -forms  $A^I$  potentials which can be used to gauge some unspecified “part” of the global  $G = SL(2, R) \times R^2$  symmetry acting on the scalar sector. We promote the constant symmetry parameters to local ones  $\alpha^A \rightarrow \Lambda^I(x) \vartheta_I^A$ , where:  $\Lambda^I(x)$  are gauge parameters and  $\vartheta \equiv \vartheta_I^A$  is the *embedding tensor* [2,3,13]. We require the theory to be invariant under the local transformations  $\delta_\Lambda \varphi = \Lambda^I \vartheta_I^A k_A \varphi$ ,  $\delta_\Lambda \tau = \Lambda^I \vartheta_I^A k_A \tau$ . It is useful to define a covariant derivative using the embedding tensor. For scalars and, in general any  $(r)_G$ -form, we define  $\mathcal{D}\eta^{(r)} = d\eta^{(r)} + \delta_\Lambda(\eta^{(r)}) = d\eta^{(r)} + A^J X_J(\eta^{(r)})$ . Where  $X_J^{(r)} = \vartheta_J^A T_A^{(r)}$ , a linear combination of the generators  $T_A$  in the representation  $(r)$  of  $G$ . We request that the covariant derivative follows a Leibnitz rule  $\mathcal{D}(XY) \sim \mathcal{D}XY + \epsilon_p X \mathcal{D}Y$ . This a non-trivial condition because in principle the  $X_J$ ’s do not follow any Jacobi identity.

Thus, we are going to require invariance under the new gauge transformations for the scalar fields, we will find that we need new couplings to the gauge 1-form fields (as usual). Repeating this procedure on the p-forms we can see we need the coupling to the  $(p+1)$ -forms, etc. One explicitly sees that the derivatives of the scalars and field strengths of any rank are covariant if we consistently assume modified gauge variations for  $p$ -form potentials where  $(p+1)$ -potentials are included as Stueckelberg terms. Most of these fields are already present in the supergravity theory or can be identified with their magnetic duals but this procedure allows us to introduce consistently the highest-rank fields (the  $d$ -,  $(d-1)$ - and  $(d-2)$ -form potentials), which are not dual to any of the original electric fields. Actually, the highest-rank potentials are related to the symmetries (Noether currents), the independent deformation parameters and the constraints that they satisfy.

A general result of the embedding tensor formalism tell us that, similarly, we need to introduce p-form potentials in the expressions of the field strengths. The field strengths will follow modified Bianchi and Ricci identities. The procedure of requesting gauge invariance becomes equivalent to the study of a tensor hierarchy of form potentials and strengths and their associated identities (the Bianchi and Ricci ones) and their possible deformations. Requesting gauge invariance amounts to the overall compatibility of the deformed Bianchi and Ricci hierarchies with the result of the appearence of strong conditions on any deformation parameters. We write in table1(right) the summary of the deformed Bianchi hierarchy. This deformation basically consist of: the substitution of the standard derivatives by newly defined ones and by adding Stueckelberg terms (“Z” terms). The Bianchi hierarchy is supplemented by a hierarchy of

Ricci-like identities, which are of the form:

$$\mathcal{D}DF^I = X_{(JK)}^I F^J \wedge F^K, \mathcal{D}DH^i = X_{Jk}^i F^J \wedge H^k, \dots \mathcal{D}DF_{(n)} = X_{(n)} F_{(2)} \wedge F_{(n-2)}.$$

The initial bosonic deformations parameters of the theory are (24 in total):  $\vartheta_I^A, Z^i, Z_i^I, Z$ . We note that if the embedding tensor  $\vartheta \rightarrow 0$  then, both, the covariant derivative and Stueckelberg couplings  $\mathcal{D} \rightarrow d$ ,  $Z \rightarrow 0$ , we recover the undeformed theory. But, on the opposite, If we take  $Z'$ s  $\rightarrow 0$  then (as can be checked by direct computation) the embedding tensor  $\vartheta \rightarrow 0$  (and then  $\mathcal{D} \rightarrow d$ , so we are dealing only with gauging deformations). No new degrees of freedom are introduced. It is implicitly assumed that the global symmetry group  $G$  is unbroken. No other massive deformation parameters, beyond  $\vartheta$  and the  $Z'$ s are permitted, this is what apparently SUSY prefers<sup>3</sup>. Finally we note the role played by the singlet  $Z$  parameter. This parameter acts as a switch of the coupling of the electrical ( $p = 1, 2, 3$ ) and magnetic ( $p > 3$ ) field strengths. We see that, if  $Z \neq 0$ , we are in presence of a twisted/derivative self-duality type condition ( $\star G \sim \frac{1}{Z} \mathcal{D}G + \dots$ ) and we are lead to a non-action theory. We will require that any deformation parameter is gauge invariant, this is trivially equivalent to impose the conditions  $\mathcal{D}\theta = \mathcal{D}Z' = 0$ <sup>4</sup>.

The gauge invariance of the theory is equivalent to the overall consistency of the Bianchi and Ricci hierarchies and imposes strong conditions on the deformations parameters  $\vartheta, Z'$ s. The general structure of the compatibility equations between the Bianchi and Ricci hierarchies can be outlined. For any n-field strength  $F_{(n)}$ , we have:

$$\mathcal{D}F_{(n)} + R_{(n)} = Z_{(n)} F_{(n+1)}, \quad \mathcal{D}DF_{(n)} = X_{(n)} F_{(2)} F_{(n)},$$

applying a covariant derivative and using  $\mathcal{D}Z_{(n)} = 0$  we get  $\mathcal{D}DF_{(n)} + DR_{(n)} = Z_{(n)} \mathcal{D}F_{(n+1)}$ . Using the assumed Ricci and Bianchi identities and imposing the Leibnitz Rule for a derivation, we get a set of algebraic identities of the form<sup>5</sup>

$$X_{(n)} F_{(2)} F_{(n)} + c_n Z_{(n-1)} F_{(2)} F_{(n)} + c_{n+1} Z_{(n)} F_{(2)} F_{(n)} + \dots = Z_{(n)} Z_{(n+1)} F_{(n+2)}.$$

As a final result, we get a set of linear (of the type  $X_{(n)} + c_n Z_{(n-1)} + c_{n+1} Z_{(n)} = 0$ ) and quadratic constraints ( $Z_{(n)} Z_{(n+1)} = 0$ ) involving the deformation parameters. The latter are, in detail,  $Z_i^I \vartheta_I^A = Z^i Z_i^I = ZZ^i = 0$ .

Furtherly, the gauge invariance of the, constant, deformation parameters (summarized by  $\mathcal{D}\vartheta = 0, \mathcal{D}Z' = 0$ ) imply a new set of conditions. The first one, the *quadratic constraint*

$$\vartheta X_J^{(ad)} - X_J^{(3)} \vartheta = 0, \quad (1)$$

and similar conditions of type  $Z_{(n)} X_J^{(n+1)} - X_J^{(n)} Z_{(n+1)} = 0$ .

We get explicit expressions for the deformed field strengths in terms of the potentials and the gauge variations of the potentials by recursion from the lowest degree, assuming the compatibility of the hierarchy.

We proceed to the deformation of the SUSY rules. At any rank, we study the modifications of the supersymmetry transformation rules of the scalars and fermion fields which are needed to ensure the closure of the local supersymmetry algebra. We replace the derivative and field strengths appearing in the fermion SUSY rules by the new covariant ones and add some new “deformation” parameters, the *fermion shifts* ( $f, k, g, h, \tilde{g}, \tilde{h}$ ). No derivatives or field strengths appear at the bosonic SUSY rules. Moreover we do not introduce any other massive deformation parameter at these bosonic form SUSY rules. This is consistent with keeping to a minimum the deformation of the bosonic tensor hierarchy (where we introduced only

<sup>3</sup> This is requested if we want to keep the modifications of the fermion SUSY rules to a minimum. Other deformation tensors will request, at least, drastic SUSY deformations.

<sup>4</sup> These conditions can be seen as the zero-degree identities in the Bianchi hierarchy

<sup>5</sup> where  $R_{(n)} = c_n F_{(2)} F_{(n-1)} + \dots, DR_{(n)} = c_n F_{(2)} DF_{(n-1)} + \dots = c_n Z_{(n-1)} F_{(2)} F_{(n)} - c_n F_{(2)} R_{(n-1)} + \dots$

the  $\vartheta, Z$ 's parameters). The commutators of any two SUSY transformation are expressions involving all the deformation parameters ( $\vartheta, Z$ 's, fermion shifts)<sup>6</sup>. We require that these deformed commutators can consistently be written in the generic form  $[\delta, \delta] = \delta_{gct} + \delta_{\Lambda} + (\text{duality})$  to lowest order in fermions. We have explicitly checked that, at any rank, we can consistently find values for the general coordinate transformation and gauge transformations ( $\Lambda$ ) so that the above general form of the commutators is obtained. The procedure allow us to find expressions for the fermion shifts in terms of the other deformation parameters (ultimately in terms of only the embedding tensor) and the scalars of the theory using the bosonic gauge constraints and a new linear constraint  $X_1^{(\tau)}\tau + X_2^{(\tau)} = 0$ <sup>7</sup>. Moreover no duality relations are needed (closure off-shell)<sup>8</sup>.

## 4 Reduction of parameters.

The effect of the linear constraints is twofold: in first place allow us to express all the deformation parameters in terms of only the embedding tensor, in second place reduces the number of independent components of this embedding tensor. These components, moreover, can be assigned to definite  $G = SL(2, R) \times R^2$  representations. From a total of 24 initial parameters we are left in a first stage with 8 Parameters ( $\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4$ ).<sup>9</sup> The representation content (weights) with respect scaling transformations is given in [19]. Let us study the effect of the quadratic constraints. The “ZZ”-type contraints ( $Z_J^A \vartheta_I^A = Z_j^I Z^j = ZZ^j = 0$ ) are automatically satisfied after imposing the previous linear constraints and the quadratic constraint given by Eq.(1). This quadratic constraint decomposes in  $G$ -irreducible representations as follows ( $Q_4, 4 \times Q_2, Q_1$ ) (see [19]). The compatibility of these set of non-linear constraints additionally imposes other secondary constraints.

The identification of the minimal set of “irreducible” quadratic constraints is a subtle issue. Let us study in some detail an interesting subcase. The  $\vartheta_1$  ( $\sim Z \sim m_{IIB}$ ) component of the embedding tensor controls the coupling of the  $G - \tilde{G}$  Bianchi identities (see the bosonic tensor hierarchy). If we set  $\vartheta_1 = 0$  (or  $Z = 0$ ) and we fix generic, non trivial, values for the other components ( $\vartheta_3, \vartheta_2 \neq 0$ ) the set of independent quadratic constraints reduces to:

$$Q_4 : (\vartheta_0^m (12\vartheta_i^4 + 5\vartheta_i^5))_4 = 0, Q_2' : \vartheta_j^4 (\vartheta_0^m T_m)_i^j = 0, Q_2'' : \vartheta_j^5 (\vartheta_0^m T_m)_i^j = 0.$$

The compatibility of these constraints implies the following secondary constraints:  $\det(T_m \vartheta_0^m) = \vartheta_0^{1,2} + \vartheta_0^{2,2} - \vartheta_0^{3,2} = 0$ ,  $\vartheta_i^4 = \lambda \vartheta_i^5$ , which can be seen as reducing the number of effective parameters. We see that: a) the list of constraints reduces to  $Q_4, Q_2', Q_2''$ , in accordance with  $E_{11}$  predictions, b) the naive counting of effective parameters is (number parameters-number constraints)  $\sharp = (3 + 2 + 2) - (1 + 1) = 3 + 2$ . We are left with a (non-linear) triplet and a doublet of parameters, they are not the originals but non linear functions of them. The conclusion is that if  $\vartheta_1 = 0$  the embedding tensor simply agree with  $\sim E_{11}$  predictions [22, 23]<sup>10</sup>.

## 5 Conclusions.

In this work we have applied the embedding-tensor formalism to the study of the most general deformations (*i.e.* gaugings and massive deformations) of maximal 9-dimensional supergravity. We have used the complete global  $SL(2, \mathbb{R}) \times \mathbb{R}^2$  symmetry of its equations of motion. We have found the constraints on

<sup>6</sup> i.e. for one of the scalars  $[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \varphi = \xi^\mu \partial_\mu \varphi + (\dots \Re(\tilde{h}), \Im(\tilde{g}), \Re(\tilde{g}) \dots)$ .

<sup>7</sup> A similar relation in appear in the application of the embedding tensor formalism in 4D SUGRA  $\vartheta_i^A P_A = 0$

<sup>8</sup> At least up degree 4

<sup>9</sup> The identification is as follows:  $\vartheta_1 \equiv \vartheta_0^5 \sim Z (\sim m_{IIB})$ ,  $\vartheta_2 \equiv \vartheta_i^4 \sim (Z^i \pm Z_i^0) (\sim (m_4, \tilde{m}_4))$ ,  $\vartheta_3 \equiv \vartheta_i^5 \sim (Z^i \pm Z_i^0) (\sim (m_{11}, m_{IIA}))$ ,  $\vartheta_4 \equiv \vartheta_0^m \sim Z_j^i (\sim (m_2 \pm m_3, m_1))$ .

<sup>10</sup> The dimensional reduction of the adjoint representation of the  $E_{11}$  algebra predicts the  $SL(2, R) \times R^+ D9$  field content, (p=7):  $\sim (\mathbf{3}, \mathbf{2})$ , (p=8):  $(\mathbf{3}, \mathbf{2})$ , (p=9):  $(\mathbf{4}, \mathbf{2}, \mathbf{2})$ .

the deformation parameters imposed by gauge and SUSY invariance (the latter imposed through the closure of the local supersymmetry algebra to lowest order in fermions). The minimal set of deformation parameters ( $8=3+2+2+1$ ) appears in agreement with Ref. [20]. We have found explicit expressions for the field strengths, gauge and SUSY rules of the deformed theory. According to the general embedding tensor framework the possible extra 7-, 8- and 9-forms, are respectively dual to the Noether currents, independent deformation tensors and irreducible quadratic constraints. Thus all the higher-rank fields have an interpretation in terms of symmetries and gaugings. We can conclude that we have satisfactorily identified the extended field content (the tensor hierarchy) of maximal 9-dimensional supergravity and, furthermore, that all the higher-rank fields have an interpretation in terms of symmetries and gaugings. In comparison with the  $E_{11}$  level decomposition [23], when comparable, the ETF gives similar results.

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